WORDS 2009 - ABSTRACTS OF INVITED LECTURES

- **M. Crochemore** (joint work with L. Ilie and L. Tinta), *The "runs" conjecture*

  Abstract: The "runs" conjecture, proposed by [Kolpakov and Kucherov, 1999], states that the number of occurrences of maximal repetitions (runs) in a string of length n, $RUNS(n)$, is at most n. We almost solve the conjecture by proving that $RUNS(n) \leq 1.029n$. This bound is obtained using a combination of theory and computer verification.

- **A. Fraenkel**, *Unbounded Iterations of Floor Functions and the Flora Game*

  Abstract: Partition the set of games into subsets CompGames and PrimGames, precipitating a new 4-pile take-away game Flora. Let $\varphi = (1+\sqrt{5})\ /\ 2$ denote the golden section. We investigate relationships between unbounded iterations of the floor function applied to various combinations of φ and φ^2. We use them to formulate an algebraic polynomial-time winning strategy for Flora, and also present recursive, arithmetic and morphic winning strategies for it. The arithmetic one is based on the Fibonacci numeration system. The four strategies differ in their computational efficiencies. We further show how to generate the sequences induced by the iterations using morphisms and and characterize them using the Fibonacci numeration system. We also exhibit an infinite array of such sequences.

- **R. Grossi**, *Combinatorics on (compressed) suffix arrays*

  Abstract: Suffix arrays can be seen as permutations of the elements in $\{1, 2, ..., n\}$, and each of them represents the sorted sequence of suffixes from a given text of length n, under the lexicographic order. Over the years, many interesting combinatorial properties have been devised for this special class of permutations: they can implicitly encode extra information, they are a well characterized subset of the $n!$ permutations, and so on. We explore and review some of these properties, showing their connection to text indexing, combinatorial pattern matching, and data compression.

- **A. Restivo**, *Burrows-Wheeler Transform and Balanced Words*

  Abstract: The investigation of the "clustering effect" of the Burrows-Wheeler transform (BWT) leads to study the words having *simple BWT*, i.e. words w over an ordered alphabet $A = \{a_1, a_2, \ldots, a_k\}$, with $a_1 < a_2 < \ldots < a_k$, such that $bwt(w)$ is of the form $a_k^{n_k} a_{k-1}^{n_{k-1}} \cdots a_1^{n_1}$, for some non-negative integers $n_1, n_2, \ldots, n_k$. We remark that, in the case of binary alphabets, there is an equivalence between words having simple BWT, the family of (circular) balanced words and the conjugates of standard words. In the case of alphabets of size greater than two, there is no more equivalence. The aim of this work is to investigate the relationships between these notions and other related ones (like, for instance, rich words) in the case of a general alphabet.

- **C. Reutenauer** (joint work with I. Assem and D. Smith), *SL_2 tilings*

  Abstract: Call SL_2 tiling a filling of the discrete plane by elements of a ring (the coefficients) in such a way that each connected submatrix has determinant 1 (similar objects have been studied by Coxeter and Conway; they call them frieze-patterns and, in the present setting, should be called partial SL_2-tilings). Given a bi-infinite word on $\{x, y\}$, interpreted as a path in the discrete plane, called the frontier, put 1s at its vertices. Then one may uniquely complete this picture in a SL_2 tiling; it turns out that the coefficients of the tiling are all positive integers; this is explained by matrix product formulas for these coefficients (the fact that these coefficients are integers also follows from the Laurent phenomenon studied by Fomin and Zelevinsky in the framework of their theory of cluster algebras). It turns

out that the geometry of the frontier implies the existence of perfect squares in the tiling: in some diagonal appear the squares of the numbers which appear in some row, or column (see below). Also, if the frontier is periodic, the sequences on each diagonal of the tiling are the coefficients of an N-rational series (that is, a positively rational series; in other words, the generating series of some rational language); an example is given by the tiling associated to the alternate frontier *...xyxyxyxy....*, where appear the Fibonacci numbers of even rank. Motivations of these constructions are the so-called frises, associated to acyclic digraphs (this construction also comes from the theory of cluster algebras). In a joint work with I. Assem and D. Smith, we showed that the sequences of the frise are all rational if and only if the digraph is a Dynkin diagram, or an affine diagram, with an acyclic orientation (recall that these diagrams play an important role in the classification of Lie algebras, and other mathematical structures).

									1	**1**	**1**	**1**		
								1	**1**	2	3	4		
								1	2	5	8	11		
								1	3	8	13	18		
				...		...		**1**	**1**	2^2	11	18	25	...
								1	2	9	5^2	41	57	
								1	3	14	39	8^2	89	
			1	**1**	**1**	**1**	**1**	**1**	4	19	53	87	11^2	
1	**1**	**1**	**1**	2	3	4	5	6	5^2	119	332	545	758	
1	2	3	4	9	14	19	24	29	121	24^2	1607	2368	3669	...
					...		...				...			

- **J. Shallit** (joint work with N. Rampersad and Z. Xu), *The computational complexity of universality problems for prefixes, suffixes, factors, and subwords of regular languages.*

  Abstract: In this paper we consider the computational complexity of the following problems: given a DFA or NFA representing a regular language L over a finite alphabet Σ, is the set of all prefixes (resp., suffixes, factors, subwords) of all words of L equal to Σ^*? In the case of testing universality for factors of languages represented by DFA's, we find an interesting connection to Cerny's conjecture on synchronizing words.

WORDS 2009 - ABSTRACTS OF ACCEPTED PAPERS

(ordered by submission number)

1. **S. V. Avgustinovich, A. Frid and T. Kamae**
Infinite permutations of lowest maximal pattern complexity
Abstract: An infinite permutation α is a linear ordering of $\mathbb{N}$. We study properties of infinite permutations analogous to those of infinite words and show some resemblances and some differences between permutations and words. In this paper, we define maximal pattern complexity $p^*_\alpha(n)$ for infinite permutations and show that this complexity function is ultimately constant if and only if the permutation is ultimately periodic; otherwise its maximal pattern complexity is at least n, and the value $p^*_\alpha(n) \equiv n$ is reached on a described family of permutations constructed with the use of Sturmian words. We also conjecture that there are no other infinite permutations of maximal pattern complexity n, so that the class described gives a characterization.

2. **H. Uscka-Wehlou**
Sturmian words with balanced construction
Abstract: In this paper we define Sturmian words with balanced construction. We formulate a fixed-point theorem for Sturmian words and analyze the set of all fixed points. The inspiration for this work came from the Kolakoski word and the general idea of self-reading sequences by Păun and Salomaa. The basis for this article is the author's earlier research on the influence of the continued fraction elements in the expansion of $a \in]0,1[\setminus \mathbf{Q}$ on the construction of runs for the upper mechanical word with slope a and intercept 0.

3. **A. Juhasz**
An application of word combinatorics to the membership problem for subsemigroups of one-relator groups
Abstract: In this work we solve the Membership Problem for certain subsemigroups in one relator groups in which the relator satisfies the small cancellation conditions $C'(1/5)$ and $T(4)$.

4. **P. Ambrož, Z. Masáková and E. Pelantová**
Substitutions fixing words associated with exchange of three intervals
Abstract: We consider words coding exchange of three intervals with permutation (3,2,1), here called 3iet words. Recently, a characterization of substitution invariant 3iet words was provided. We study the opposite question: what are the morphisms fixing a 3iet word? We reveal a narrow connection of such morphisms and morphisms fixing Sturmian words using the new notion of amicability.

5. **Y. Bugeaud, D. Krieger and J. Shallit**
Morphic and Automatic Words: Maximal Blocks and Diophantine Approximation
Abstract: Let $\mathbf{w}$ be a morphic word over a finite alphabet Σ, and let Δ be a nonempty subset of Σ. We study the behavior of maximal blocks consisting only of letters from Δ in $\mathbf{w}$, and prove the following: let (i_k, j_k) denote the starting and ending positions, respectively, of the k'th maximal Δ-block in $\mathbf{w}$. Then $\limsup_{k\to\infty}(j_k/i_k)$ is algebraic if $\mathbf{w}$ is morphic, and rational if $\mathbf{w}$ is automatic. As a result, we show that the same conclusion holds if (i_k, j_k) are the starting and ending positions of the k'th maximal zero block, and, more generally, of the k'th maximal x-block, where x is an arbitrary word.

This enables us to draw conclusions about the irrationality exponent of automatic and morphic numbers. In particular, we show that the irrationality exponent of automatic (resp., morphic) numbers belonging to a certain class that we define is rational (resp., algebraic).

6. **P. Ambrož, A. Frid, Z. Masáková and E. Pelantová**
Enumeration of 3iet factors
Abstract: We consider exchange of three intervals with permutation $(3, 2, 1)$. The aim of this paper is to count the cardinality of the set $3iet(N)$ of all words of length N which appear as factors in infinite words coding such transformations. We use the strong relation of 3iet words and words coding exchange of two intervals, i.e., Sturmian words. The known asymptotic formula $\#2iet(N)/N^3 \sim 1/\pi^2$ for the number of Sturmian factors allows us to find bounds $1/(3\pi^2) + o(1) \leq \#3iet(N)/N^4 \leq 2/\pi^2 + o(1)$.

7. **D. Merlini and R. Sprugnoli**
Algebraic aspects of some Riordan arrays related to binary words avoiding a pattern
Abstract: We consider some Riordan arrays related to binary words avoiding a pattern which can be easily studied by means of an *A-matrix* rather than their *A-sequence*. Both concepts allow us to define every element as a linear combination of other elements in the array; the A-sequence is unique while the A-matrix is not. However, for the problems considered in the present paper, we show that the A-matrix approach is more convenient. We provide explicit algebraic generating functions for these Riordan arrays and obtain many statistics on the corresponding languages.

8. **F. Blanchet-Sadri and B. Shirey**
Periods and Binary Partial Words
Abstract: A well known and unexpected result of Guibas and Odlyzko states that the set of all periods of a word is independent of the alphabet size (alphabets with one symbol are excluded here). Recently, Blanchet-Sadri and Chriscoe extended this fundamental result to words with one "do not know" symbol also called partial words with one hole. They showed that for every partial word u with one hole, there exists a partial word v over the alphabet $\{0, 1\}$ sharing the same length and same sets of periods and weak periods as u, and satisfying $H(u) \supset H(v)$ where $H(u), H(v)$ denote the sets of holes of u, v. In this paper, we provide an algorithm that given a *nonspecial* partial word u with an arbitrary number of holes, computes a partial word v as described. A World Wide Web server interface at `www.uncg.edu/cmp/research/bintwo` has been established for automated use of the program.

9. **M.Arfi, B. Ould M Lemine, C. Selmi**
Mixed strategies and closed sets of infinite words
Abstract: We deal in this paper with the notion of mixed strategy on a finite alphabet A. We show that a mixed strategy on A allows us to define a pseudo-probability on the family of closed sets in the topological space of infinite words on A.

10. **L. Balková, E. Pelantová and Š. Starosta**
Palindromes in infinite ternary words
Abstract: We study infinite words $\mathbf{u}$ over an alphabet $\mathcal{A}$ satisfying the property

$$\mathcal{P}: \qquad \mathcal{P}(n) + \mathcal{P}(n+1) = 1 + \#\mathcal{A} \quad \text{for any } n \in \mathbb{N},$$

where $\mathcal{P}(n)$ denotes the number of palindromic factors of length n occurring in the language of $\mathbf{u}$. We study also infinite words satisfying a stronger property

$$\mathcal{PE}: \quad \text{every palindrome of } \mathbf{u} \text{ has exactly one palindromic extension in } \mathbf{u}.$$

For binary words, the properties $\mathcal{P}$ and $\mathcal{PE}$ coincide and these properties characterize Sturmian words, i.e., words with the complexity $\mathcal{C}(n) = n + 1$ for any $n \in \mathbb{N}$. In this paper, we focus on ternary infinite words with the language closed under reversal. For such words $\mathbf{u}$, we prove that if $\mathcal{C}(n) = 2n+1$ for any $n \in \mathbb{N}$ then $\mathbf{u}$ satisfies the property $\mathcal{P}$ and moreover $\mathbf{u}$ is rich in palindromes. Also a sufficient condition for the property $\mathcal{PE}$ is given. We construct a word demonstrating that $\mathcal{P}$ on a ternary alphabet does not imply $\mathcal{PE}$.

11. **T. Kärki, A. Lacroix and M. Rigo**
On the recognizability of self-generating sets

12. **E. Duchêne and M. Rigo**
Invariant Games
Abstract: In the context of 2-player removal games, we define the notion of invariant game for which each allowed move is independent of the position it is played from. We present a family of invariant games which are variations of Wythoff's game. The set of P-positions of these games are given by a pair of complementary Beatty sequences related to the irrational quadratic number $\alpha_k = (1; \overline{1, k})$. We also provide a recursive characterization of this set. This paper illustrates some applications of words combinatorics to combinatorial game theory.

13. **M. Rao**
Last Cases of Dejean's Conjecture
Abstract: Dejean conjectured that the repetition threshold for a k-letter alphabet (with $k \geq 2$) is $\frac{k}{k-1}$, except for $k \in \{3, 4\}$. Dejean's conjecture has already been proved for $k \leq 14$ and for $k \geq 27$. We present here a proof for $8 \leq k \leq 34$.

14. **R. Dabrowski and W. Plandowski**
Complete characterization of zero-expressible functions
Abstract: In [KaMiPl1] the family of expressible relations is introduced. We describe an intersection of the family of expressible relations and another natural family of relations. This is the first result of this kind existing in the litterature. To obtain it we extend a tool from [KaMiPl1] for proving nonexpressibility of languages to a tool for proving nonexpressibility of relations.

15. **V. Vajnovszki**
A new Euler-Mahonian constructive bijection
Abstract: Using generating functions, MacMahon proved in 1916 the remarkable fact that the major index has the same distribution as the inversion number for multiset permutations, and in 1968 Foata gave a constructive bijection proving MacMahon's result. Since then, many refinements have been derived, consisting of adding new constraints or new statistics. Here we give a new simple constructive bijection between the set of permutations with a given number of inversions and those with a given major index. We introduce a new statistic, mix, related to the Lehmer code, and using our new bijection we show that the bistatistic (mix, inv) is Euler-Mahonian. Finally we introduce the McMahon code for permutations which is the major-index counterpart of Lehmer code and show how the two codes are related.

16. **F. Fiorenzi and P. Ochem**
More on generalized repetition thresholds
Abstract: The *repetition threshold* introduced by Dejean and Brandenburg is the smallest real number α such that there exists an infinite word over a k-letter alphabet that avoids β-powers for all $\beta > \alpha$. Ilie, Ochem, and Shallit generalized this concept to include the

lengths of the avoided words. We give a lower and an upper bound on this generalized repetition threshold.

17. **S. Brocchi, A. Frosini, R. Pinzani, S. Rinaldi**
On the tiling recognizability of L-convex polyominoes
Abstract: A polyomino is said to be L-convex if any two of its cells can be connected by a path entirely contained in the polyomino, and having at most one change of direction. In this paper, answering to a problem posed by Castiglione and Vaglica, we prove that the class of L-convex polyominoes is tiling recognizable. To reach this goal, first we express the L-convexity constraint in terms of a set of independent properties, then we show that each class of convex polyominoes having one of these properties is tiling recognizable.

18. **G. Richomme, K. Saari and L. Q. Zamboni**
Abelian Properties of Words (Extended abstract)
Abstract: We say that two finite words u and v are abelian equivalent if and only if they have the same number of occurrences of each letter, or equivalently if they define the same Parikh vector. In this paper we investigate various abelian properties of words including abelian complexity, and abelian powers. We study the abelian complexity of the Thue-Morse word and the Tribonacci word, and answer an old question of G. Rauzy by exhibiting a class of words whose abelian complexity is everywhere equal to 3. We also investigate abelian repetitions in words and show that any infinite word with bounded abelian complexity contains abelian k-powers for every positive integer k.

19. **O. Klíma**
Piecewise Testable Languages via Combinatorics on Words
Abstract: A regular language L over an alphabet A is called piecewise testable if it is a finite boolean combination of languages of the form $A^*a_1A^*a_2A^*\dots A^*a_\ell A^*$, where $a_1,\dots,a_\ell \in A$, $\ell \geq 0$. An effective characterization of piecewise testable languages was given in 1972 by Simon who proved that a language L is piecewise testable if and only if its syntactic monoid is $\mathcal{J}$-trivial. Nowadays there exist several proofs of this result based on various methods from algebraic theory of regular languages. Our contribution adds a new purely combinatorial proof.

20. **D. Jamet, G. Paquin, G. Richomme and L. Vuillon**
On the fixed points of the iterated pseudopalindromic closure
Abstract: First introduced in the study of the Sturmian words, the iterated palindromic closure was recently generalized to pseudopalindromes. This operator allows one to construct words with infinitely many pseudopalindromic prefixes, called pseudostandard words. We provide here several combinatorial properties of the fixed points under the iterated pseudopalindromic closure.

21. **F. D'Alessandro, B. Intrigila and S. Varricchio**
On some counting problems for semi-linear sets
Abstract: Given a set X of $\mathbb{N}^t$ or $\mathbb{Z}^t$, we can associate with X a map $f_X : \mathbb{N}^t \longrightarrow \mathbb{N}$ which returns, for every $(n_1,\dots,n_t) \in \mathbb{N}^t$, the number $f_X(n_1,\dots,n_t)$ of all vectors $x \in X$ such that, for every $i = 1,\dots,t$, $|x_i| \leq n_i$. This map is called the *growth function* of X. The main result of this paper is that the growth map of a semi-linear set of $\mathbb{N}^t$ or $\mathbb{Z}^t$ is a box spline. By using this result and some theorems on semi-linear sets, we give a new proof of combinatorial flavour of a well-known theorem by Dahmen and Micchelli on the counting map of a system of Diophantine linear equations.

22. **X. Provençal**
Non-Convex Words

Abstract: Using a combinatorial characterization of digital convexity based on words, one defines the language of convex words. The complement of this language form an ideal whose minimal elements, with respect to the factorial ordering, appear to have a particular combinatoral structure very close to the Christoffel words. In this paper, those words are completely characterized as those of the form uw^kv where $k \geq 1$, $w = u \cdot v$ and u, v, w are Christoffel words.

23. **S. Puzynina**
On p-adic avoidance of words
Abstract: In this paper a new modification of the notion of avoidance of words is introduced. We say, that a word w p-adically avoids a word u, if u does not belong to the set of all words whose symbols occur in w at positions which constitute arithmetical progressions with common differences equal to integer powers of p. We study the language of words over the binary alphabet $\Sigma = \{a, b\}$, p-adically avoiding the word consisting of k successive letters b.

24. **A. Blondin Massé, S. Brlek, A. Garon and S. Labbé**
Palindromes and local periodicity
Abstract: In this paper we consider several types of equations on words, motivated by the attempt of characterizing the class of polyominoes that tile the plane by translation in two distinct ways. Words coding the boundary of these polyominoes satisfy an equation whose solutions are in bijection with a subset of the solutions of equations of the form $AB\tilde{A}\tilde{B} \equiv XY\tilde{X}\tilde{Y}$. It turns out that the solutions are strongly related to local periodicity involving palindromes and conjugate words.

25. **S. Crespi Reghizzi and D. Mandrioli**
Algebraic properties of structured context-free languages: old approaches and novel developments
Abstract: The historical research line on the algebraic properties of structured CF languages initiated by McNaughton's Parenthesis Languages has recently attracted much renewed interest with the Balanced Languages, the Visibly Pushdown Automata (VPDA) languages, the Synchronized Languages, and the Height-deterministic ones. Such families preserve to a varying degree the basic algebraic properties of Regular languages: boolean closure, closure under reversal, under concatenation, and Kleene star. We prove that the VPDA family is strictly contained within the Floyd Grammars (FG) family historically known as operator precedence. Languages over the same precedence matrix are known to be closed under boolean operations, and are recognized by a machine whose pop or push operations on the stack are purely determined by terminal letters. We characterize VPDA's as the subclass of FG having a peculiarly structured set of precedence relations, and balanced grammars as a further restricted case. The non-counting invariance property of FG has a direct implication for VPDA too.

26. **F. Blanchet-Sadri, R. Mercas and K. Wetzler**
The Three-Squares Lemma for Partial Words with One Hole
Abstract: Partial words, or sequences over a finite alphabet that may have do not know symbols or holes, have been recently the subject of much investigation. Several interesting combinatorial properties have been investigated such as the periodic behavior and the counting of distinct squares in partial words. In this paper, we extend the three-squares lemma on words to partial words with one hole. This result provides special information about the squares in a partial word with at most one hole, and puts restrictions on the positions at which periodic factors may occur, which is in contrast with the well known periodicity lemma of Fine and Wilf.

27. **M. Anselmo and M. Madonia**
Some results on unary unambiguous automata
Abstract: The paper focuses on deterministic and unambiguous finite automata (DFA's and UNFA's respectively for short) in the case of a one-letter alphabet. We present a structural characterization of unary UNFA's and some considerations relating minimal UNFA's with minimum DFA's recognizing a given unary language. Then we establish a correspondence between pairs of UNFA's recognizing a unary language and its complement respectively, and the disjoint covering systems of number theory. It allows us to provide some conditions relating the number of final states and the lengths of cycles in an UNFA recognizing a unary language with the same parameters in an UNFA recognizing its complement.

28. **E. Vaslet**
Bounds for the generalized repetition threshold
Abstract: The notion of repetition threshold, which is the object of Dejean's conjecture (1972), was generalized by Ilie, Ochem, and Shallit in 2005, to take into account not only the exponent of the (non-)avoided repetitions, but also their length. In this paper, we will give some bounds for this threshold $RT(k, l)$.

29. **J.-P. Borel**
Digital Straight Lines and Billiard Words
Abstract: Standard Billiard Words are the coding words of half-lines with positive slope, and starting from the origin. In this case the crossed pixels are the unit squares. We use in this presentation other kinds of pixels and try to ask to the following question : is there an automatic way, i.e., a finite transducer, which can be used to compute the new coding word, starting from the corresponding Standard Billiard Word ? Up to technical conditions, this can be done if and only if the pixels are convex polyhedrons, whose vertices have rational coordinates. In this case, the same transducer can be used for any half-line with positive slope, i.e., any infinite Billiard Word, and also for finite Billiard Words.

30. **L. Breveglieri, S. Crespi Reghizzi and M. Goldwurm**
Efficient recognition of trace languages defined by repeat-until loops
Abstract: A sequence of operations (or instructions) may be validly reordered, provided that only pairs of independent operations are commuted. Focusing on a program scheme, idealized as a local DFA recognizing a set of strings, we consider the following decision problem: is a given string a valid permutation of a recognized one? Within the framework of trace theory, this is the word membership problem for rational trace languages. Existing general algorithms, although time-polynomial, have unbounded degree related to some properties of the dependence graph. Here we present two original linear-time solutions. A straightforward algorithm is suitable for any local DFA such that any two successors of an operation are dependent or not mutually reachable. The second approach is currently restricted to nested repeat-until loops. Using integer compositions to represent loop iterations, the algorithm constructs the loop nesting syntax tree by exploiting newly introduced functions on integer compositions. The result may be relevant for checking dependencies of rescheduled programs on parallel processors.

31. **S. Lombardy and J. Sakarovitch**
Radix cross-sections for length morphisms
Abstract: We prove that the radix cross-section of a rational set for a length morphism, and more generally for a rational function from a free monoid into N (the set of natural integers), is rational, a property that does not hold any more if the image of the function is a subset of a free monoid with two or more generators.

32. **S. Julia and T. Vinh Duc**
Families and ω-ambiguity removal
Abstract: We consider the following open problem: let L be a rational language, how to decide whether L^ω is generated by an ω-code? We investigate languages whose ambiguity and/or ω-ambiguity are minimal. To do this, we introduce a notion of family over relations fulfilled by words. If L is finite, the number of families is finite. Each of them produces its proper set of incompatible prefixes. In case of a unique family, this leads to two twin languages that are candidates to be ω-generator ω-code if some exist.

33. **S. Holub and J. Kortelainen**
On partitions separating two words
Abstract: Let Σ be a finite alphabet, $u, v \in \Sigma^+$ and $L \subseteq \Sigma^+$. Call the $(L, \Sigma^+ \setminus L)$ a closed partition (of Σ^+) separating the words u and v if (i) $u \in L$ (ii) $v \in L \setminus \Sigma^+$ and (iii) both L and $\Sigma^+ \setminus L$ are semigroups (or closed), i.e., $L = L^+$ and $\Sigma^+ \setminus L = (\Sigma^+ \setminus L)^+$. A new proof is provided to the fact that there exists a closed partition separating two words if and only if the words do not commute. A geometric interpretation of the separating languages in case that the Parikh images of the words are linearly independent is given. We also show that the two separating languages can be regular, and construct the corresponding automaton.

34. **M. Dominguez and T. Noll**
A Specific Extension of Christoffel-Duality to a Certain Class of Sturm Numbers and their Characteristic Words
Abstract: This paper is the result of a further elaboration of previous works on the junction of two differentiated disciplines: algebraic combinatorics on words and algebraic musical scale theory. Algebraic theory of scales can benefit from the new lenguage and procedures of combinatorics, but this benefit can also be reciprocal. Musical facts motivate combinatorial questions that can eventually flow into new results in combinatorics of words. That was the case of [Dominguez09] and [Thomas08], where the natural interest on rotations of scales (musical modes) led to new characteritations of special standard morphisms and also to an extension of the Sturmian involution. The aforementioned feedback is also present in this paper, where we try to solve the following question: given a Christoffel word, which is a prefix of a Sturmian word of slope g, is there a *natural* way of determining the slope g^* of a Sturmian word which prefix is the corresponding Christoffel dual? In that case, which is the meaning of that *natural dual slope*, and what consequences can we get in terms of Sturmian morphisms?

35. **I. A. Mikhailova**
Pattern avoidance by antichains
Abstract: We prove that for any avoidable pattern p, there exists an infinite set M of words over a suitable finite alphabet depending only on the number of variables in p such that each word in M avoids both p and any other word from M.

36. **D. Clampitt and T. Noll**
Pairwise Well-formed Words
Abstract: Pairwise well-formed words are words on a three-letter alphabet whose pairwise projections are conjugates of Christoffel words. In the infinite case, the projections are Sturmian. A construction for producing infinite pairwise well-formed sequences is given, and properties, such as balance, are demonstrated for both classes.